**Binary string generating algorithm game for two people**

**(as told in Ivars Peterson's "Islands of Truth")**

Somebody **A** writes down a secret string of six 0s and 1s in any order.

Somebody else **B** writes down four 'random' six-strings. **A** scores them giving one mark for every digit that is correctly placed. This is where it departs from guessing games. The two highest scoring strings (or a choice from the highest scoring strings) are then written down twice to make four strings, e.g. if 000111 and 110101 were the highest scoring strings the next display would be:

000111

110101

000111

110101

The first two are cut at the same arbitrary point, swapped and re-spliced together to make four new strings of six. The second two are cut at another point and similarly swapped and re-spliced.

e.g. from the four strings above we could have:

000`101

110`111

0`10101

1`00111

The new strings are scored by **A** and the whole process is repeated.

He says the process leads automatically to the correct string, but in trials I found it did not necessarily. Try it a few times to see what goes wrong. How can the process be tweaked so that it works? I contacted Ivars to query this and ask for its source. He did not know its source and said that some generating algorithms do not always work for small strings, which this is, without more rules. (He did not say this in his original book).

Good luck!

**DUAL GAMES ON HEXAGONAL GRIDS (use a three by three grid)**

**SHANNON SWITCHING GAME**

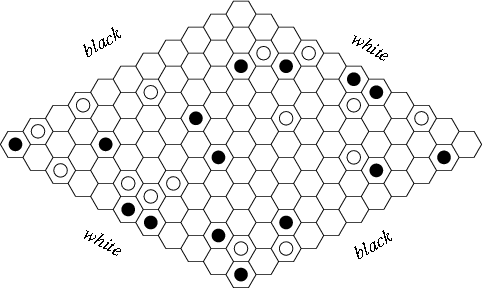
**One player called SHORT aims to traverse a connected graph from vertex A to vertex B; one 'go' consists colouring one edge. The other player called CUT aims to prevent this by removing edges: one 'go' consists of removing one edge. SHORT wins if the connection from A to B is made. CUT wins if the connection from A to B is prevented.**

**THE DUAL**

**This game is like Hex except that one player called JOIN aims to join two opposite edges of the grid (use a small one to start with) and the other called CUT aims to cut across the path.**



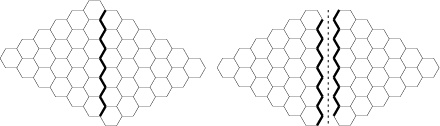
**Game of Hex**



Hex is a two-player [game](http://mathworld.wolfram.com/Game.html) invented by Piet Hein in 1942 while a student at Niels Bohr's Institute for Theoretical Physics, and subsequently and independently by John Nash in 1948 while a mathematics graduate student at Princeton. The game was originally called Nash or John, with the latter name at the same time crediting its inventor and referring to the fact that it was frequently played on the tiled floors of bathrooms (Gardner 1959, pp. 74-75). The name Hex was invented in 1952, when a commercial version was issued by the game company Parker Brothers.

Hex is played on a diamond-shaped board made up of hexagons. The game is usually played on a boards of size 11 on a side, for a total of 121 hexagons, as illustrated above. In the game, one player plays white pieces, while the other plays black, with play alternating between players and placement only allowed on unoccupied hexagons. Alternate sides of the board are designated white and black as shown above, and the goal of the game is to complete a chain of pieces between one player's two sides. The game cannot end in a [draw](http://mathworld.wolfram.com/Draw.html) since no chain can be completely blocked except by a complete chain of the opposite color.

In 1949, Nash showed using a reductio ad absurdum proof that there is always a winning strategy for the first player on an n×nboard of any size. However, this provides only an existence proof. The win/lose status has been determined for every move in 7×7hex (Hayward). A winning strategy is known for 8×8and 9×9boards assuming a first play at the center of the board (Yang), but not larger square boards. C. F. Shannon and E. F. Moore built a hex-playing machine that associated a two-dimensional electrical charge distribution with any given Hex position. This machine then made decisions based on properties of the corresponding potential field (Shannon 1953).



For play on a n×(n+1)board, the second player, playing the shorter direction, can always win by playing a mirror image move, as illustrated above (Gardner 1959).

A modified version changes the rules so that the first player to form a chain *loses*. For this variant, there is a winning strategy for the first player if there is an even number of cells on each side; otherwise, there is a winning strategy for the second player (Gardner 1959, p. 78).

**Crab Canon from Douglas Hofstadter: Goedel, Escher, Bach**

Achilles and the Tortoise happen upon each other in the park one day while strolling.

Tortoise: Good day, Mr. A.

Achilles: Why, same to you.

Tortoise: So nice to run into you.

Achilles: That echoes my thoughts.

Tortoise: And it's a perfect day for a walk. I think I'll be walking home

soon.

Achilles: Oh, really? I guess there's nothing better for you than walking.

Tortoise: Incidentally, you're looking in fine fettle these days, I must

say.

Achilles: Thank you very much.

Tortoise: Not at all. Here, care for one of my cigars?

Achilles: Oh, you are such a philistine. In this area, the Dutch contribu-

tions are of markedly inferior taste, don't you think?

Tortoise: I disagree, in this case. But speaking of taste, I finally saw that

Crab Canon by your favorite artist, M.C. Escher, in a gallery the other

day, and I fully appreciate the beauty and ingenuity with which he

made one single theme mesh with itself going both backwards and

forwards. But I am afraid I will always feel Bach is superior to Escher.

Achilles: I don't know. But one thing for certain is that I don't worry about

arguments of taste. De gustibus non est disputandum.

Tortoise: Tell me, what's it like to be your age? Is it true that one has no

worries at all?

Achilles: To be precise one has no frets.

Tortoise: Oh, well, it's all the same to me.

Achilles: Fiddle. It makes a big difference, you know.

Tortoise: Say, don't you play the guitar?

Achilles: That's my good friend. He often plays, the fool. But I myself

wouldn't touch a guitar with a ten-foot pole.

(Suddenly the Crab, appearing from out of nowhere, wanders up ex-

citedly, pointing to a rather prominent black eye.)

Crab: Hallo! Hullo! What's up? What's new? You see this bump, this

from Warsaw - a collosal bear of a man - playing a lute. He was three

meters tall, if I'm a day. I mosey on up to the chap, reach skyward and

manage to tap him on the knee, saying, "Pardon me, sir, but you are

Pole-luting our park with your mazurkas." But WOW! he had no sense

of humor - not a bit, not a wit - and POW! - he lets loose and belts me

one, smack in the eye! Were it in my nature, I would crab up a storm,

but in the time-honored tradition of my species, I backed off. After all,

when we walk forwards, we move backwards. It's in our genes, you

know, turning round and round. That reminds me - I've always

wondered, "which came first - the Crab or the Gene?" That

is to say, "Which came last - the Gene, or the Crab?" I'm always

turning things round and round, you know. It's in our genes, after

all. When we walk backwards we move forwards. Ah me, oh my!

I must lope along on my merry way - so off I go on such a fine day.

Sing "ho!" for the life of a Crab! TATA! Ole!

(And he disappears as suddenly as he arrived.)

Tortoise: That's my good friend. He often plays, the fool. But I myself

wouldn't touch a ten-foot Pole with a guitar.

Achilles: Say, don't you play the guitar?

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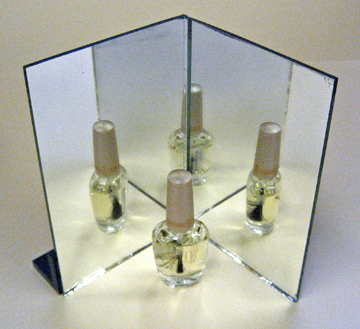
Achilles: Good day, Mr. A.

**Interesting inverses**

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| **Socks and shoes**  The inverse of 'put on my socks, put on my shoes' is 'take off my shoes, take off my socks'.  The inverse of 'add 2 and multiply by 4' is 'divide by 4 and subtract 2'.  The graphs of a linear function and its inverse are reflections of each other across the line *y = x*. Why?  Is this true for all functions?  Make up 'think of a number' problems where the answer is 'the number you first thought of'. |
| **Adding from?**  Inverse operations: e.g. the inverse of 'subtract 3' is 'add 3':  what action undoes 'subtract from 13'?  what action undoes 'divide into 24'? |
| **Flipping cups**  You have N cups, all pointing upwards initially.  On any move, you can turn over any M of them.  Is it possible to have all N cups point downwards?  *(there is further information about this question on a supplementary sheet)* |
| **Translations of translations**  X means 'translate from English into French'; Y means 'translate from French into German'.  So X-1 means what? And Y-1?  What do these mean?  XY; YX; X-1 Y-1; Y-1X-1 ; X-1 Y; etc.  XX-1 = ?  Does (YX)-1 = X-1 Y-1  Do geometrical translations work the same way? |

|  |
| --- |
| **An arrow operation**  Assume there is an arrow operation ⌂ such that ⌂ (it may help to draw these)  Is this commutative?  Is there an identity - a vector so that, for any : ⌂ and  ⌂ ?  Are there inverses - a vector so that ⌂ gets you back where you started?  Is the operation associative? |
| **LCMs**  Assume a \* b = lcm (a,b)  What is the identity? Are there any inverses? |
| **More stuff from Budden: Fascination for Groups**  Let O be a fixed point and for points on the plane such as A, B, ... etc. there is the operation:  A \* B = P so that OAPB is a parallelogram  Show that this operation is associative (i.e. (A\*B)\*C = A\*(B\*C)) and investigate its identity and inverses.  Can the same be said of this operation: A\*\*B = P so that OABP is a parallelogram? (Budden p.70 q. 24) |
| **Rational inverses**  Find inverse functions for: ; ; ; ; ; *(if you have been to previous institutes you may have met these before with a differently posed question - hints can be given)*  For what values of a,b,c,d does f(x) = f-1(x)? f2(x) = f-1(x)? f3(x) = f-1(x)? etc. |

**Reflections of reflections**



Imagine two parallel mirrors with a (non-symmetrical) object in between:

Generate the resulting images.

Call reflections in one of the mirrors *a* and the other *b* so that you are generating reflections which can be symbolised by expressions like *a2 , aba,* etc.Investigate any strings of reflections that produce the same result.

See Cabri and Geogebra files

For non-parallel mirrors (see pictures) what can happen? Investigate any equivalent combinations of reflections.

What if three mirrors are arranged in an equilateral triangle, or any triangle?

You may like to read the excerpt from 'Mister God this is Anna' by Fynn.

**Symbolic Logic**

(from Budden p.63)

A and B are two sets; A' is the complement of A (i.e. the ¬A, or 'not A'). So (A')' = A.

∩ means intersection; U means union; Φ means empty set; for this session, Σ stands for the universal set which strictly speaking should be defined as 'the set of all .....', or we should be told: A U A' = Σ .

Do the operations ∩ and U have identities and inverses?

Explore various cases, e.g. (A' ∩ B)' U C, including any special cases.

Explore the behaviour of: (A' ∩ B) U (A ∩ B'). Does this operation have an identity and inverses? This called 'the symmetric difference'.

We are going to give this operation a symbol: Δ so that A Δ B **=** (A' ∩ B) U (A ∩ B')

Find, and illustrate in Venn diagrams: A Δ (A ∩ B); A Δ (A U B); A Δ B Δ C

Explore special cases.

**True/false**

There are four doors: X,Y,Z,W. At least one of them leads you to safety but the others lead to a devouring dragon. Eight priests give you advice, A,B,C,D,E,F,G,H. Each of them is either a truthteller or a liar. They tell you:

A: X is a good door

B: At least one of the doors Y and Z is good

C: A and B are both telling the truth

D: X and Y are both good doors

E: X and Z are both good doors

F: Either D or E is telling you the truth

G: If C is telling you the truth, then so is F

H: If G and I are both telling you the truth, then so is A.